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MATHEMATICAL SIMULATION OF HEAT AND MASS TRANSFER, PHASE CONVERSIONS, AND SHRINKAGE FOR OPTIMIZATION OF THE PROCESS OF DRYING OF THERMOLABILE MATERIALS

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A mathematical model and a numerical method for calculation of the processes of heat and mass transfer, phase conversions, and shrinkage in drying of colloidal capillary-porous bodies are presented. The model and method allow one to conduct polyvariant calculation of the fields of temperature, volumetric concentration, and pressure in each of the components of the bound substance. A method of dehydration of thermolabile materials that makes it possible to minimize the time of drying and provide savings of energy and resources is presented.

Drying of disperse materials, which is widely used in different branches of industry and agriculture, requires great energy and material consumption. In connection with this, a modern trend in the development of drying technologies is the saving of energy and resources and enhancement of the dehydration process with the high quality of the finished product being preserved.

The possibility of enhancement of the process of drying of thermolabile materials due to increase in the heat-carrier temperature is limited, since the effect of elevated temperatures on these materials leads to their destruction or irreversible qualitative changes. In dehydration of thermolabile materials, at each instant of time the body temperature must be lower than some maximum permissible value T^*. If the maximum permissible temperature is relatively low, dehydration of thermolabile materials at a constant temperature of the drying agent $T_{\text{sur}} = T^*$ is impractical. This is due to the fact that at low temperatures the time of the drying process increases greatly, its efficiency decreases, and the quality of the product can deteriorate. Therefore, development of a highly efficient technology of drying of thermolabile materials is related to the necessity of varying the heat-carrier temperature during drying.

Most thermolabile materials are colloidal capillary-porous bodies the volume of which can decrease severalfold in drying. Shrinkage phenomena can greatly affect the dynamics of the processes of heat and mass transfer and phase conversions. In mathematical models used for describing the processes of drying [1–5], shrinkage of material is usually disregarded.

In the present paper, we state the mathematical model and the numerical method of calculation of heat and mass transfer, phase conversions, and deformation in dehydration of porous bodies, on the basis of which a new method of drying of thermolabile materials is developed. This method provides a minimum time of the process and decrease in energy losses at the given values of the maximum permissible temperature and the maximum temperature of the drying agent.

Mathematical Model. We consider an isotropic, colloidal, capillary-porous body that deforms as a result of heat and mass transfer processes, phase conversions, or under the action of outer forces. In Cartesian coordinates, we denote x_1, x_2, and x_3 in terms of $x_i(t) = x_i[t, x_1(0), x_2(0), x_3(0)]$ $(i = 1, 2, 3)$, i.e., the coordinates of the point of this body at a time instant t, which at $t = 0$ had the coordinates $x_i(0)$. The relation between $x_i(t)$ and $x_i(0)$ can be expressed in terms of the components $u_i(t) = u_i[t, x_1(0), x_2(0), x_3(0)]$ $(i = 1, 2, 3)$ of the displacement vector:

$$x_i(t) = x_i(0) + u_i(t) . \tag{1}$$

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The components of the displacement vector are connected with the components of the tensor of deformations in the Cartesian coordinates by the relations $\varepsilon_{ij} = \partial u_i/\partial x_j$ (i, j = 1, 2, 3) [6].

In the considered body at the time instant t we distinguish an elementary parallelepiped whose volume is $\Delta V(t) = \Delta x_1(t)\Delta x_2(t)\Delta x_3(t)$. According to (1), the length of the parallelepiped edge $\Delta x_1(t)$ can be determined as follows:

$$\Delta x_1(t) = x_i\,[t, x_1(0) + \Delta x_1(0), x_2(0), x_3(0)] - x_i\,[t, x_1(0), x_2(0), x_3(0)] =$$

$$= x_1(0) + \Delta x_1(0) + u_1\,[t, x_1(0) + \Delta x_1(0), x_2(0), x_3(0)] - x_1(0) - u_1(0) =$$

$$= \Delta x_1(0)\left\{1 + \frac{\partial u_1\,[t, x_1(0), x_2(0), x_3(0)]}{\partial x_1(t)}\right\} = \Delta x_1(0)\,[1 + \varepsilon_{11}(t)]\,. \qquad (2)$$

Then we find the relative volumetric strain ε_V in the vicinity of this point of the body as

$$\varepsilon_V(t) = \frac{\Delta V(t) - \Delta V(0)}{\Delta V(0)} = [1 + \varepsilon_{11}(t)]\,[1 + \varepsilon_{22}]\,[1 + \varepsilon_{33}(t)] - 1\,. \qquad (3)$$

The expression for the rate of time-variation of the relative volume of the body element follows from relations (2) and (3); this expression is required to take into account the influence of deformation on the processes of heat and mass transfer:

$$\frac{1}{\Delta V(t)}\frac{\partial \Delta V(t)}{\partial t} = \frac{1}{1 + \varepsilon_V(t)}\frac{\partial \varepsilon_V(t)}{\partial t} = \frac{1}{1 + \varepsilon_{11}}\frac{\partial \varepsilon_{11}}{\partial t} + \frac{1}{1 + \varepsilon_{22}}\frac{\partial \varepsilon_{22}}{\partial t} + \frac{1}{1 + \varepsilon_{33}}\frac{\partial \varepsilon_{33}}{\partial t}\,. \qquad (4)$$

Expressions for the components of the strain tensor, provided the fields of temperature T, volumetric concentrations U_ψ, or specific mass contents ω_ψ of the bound substances — liquid (ψ = liq), vapor (ψ = v), and air (ψ = a) — and outer stresses are specified, can be obtained based on the equations of thermoconcentration strain [7]. For the case of elastic deformation of a porous body, when mass forces and dynamic effects of deformation that manifest themselves only in intense nonstationary processes of heating and drying of thin-walled bodies can be neglected, the equation of momentum transfer in the displacements in the Cartesian coordinates x_i (i = 1, 2, 3) can be written in the form

$$\frac{\partial}{\partial x_1}\left(G\frac{\partial u_i}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(G\frac{\partial u_i}{\partial x_2}\right) + \frac{\partial}{\partial x_3}\left(G\frac{\partial u_i}{\partial x_3}\right) + \frac{\partial}{\partial x_i}\left[G_1\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}\right)\right] +$$

$$+ \frac{\partial}{\partial x_1}\left(G\frac{\partial u_1}{\partial x_i}\right) + \frac{\partial}{\partial x_2}\left(G\frac{\partial u_2}{\partial x_i}\right) + \frac{\partial}{\partial x_3}\left(G\frac{\partial u_3}{\partial x_3}\right) - \frac{\partial}{\partial x_i}\,[N\,(2G + 3G_1] = 0\,. \qquad (5)$$

Here G and G_1 are the Lamé coefficients, $G = E_{\text{el}}[2(1 + \nu_{\text{v}})]$, $G_1 = E_{\text{el}}\nu_{\text{v}}/[(1 - 2\nu_{\text{v}})(1 + \nu_{\text{v}})]$, and N is the thermoconcentration function that determines variation of the specific volume of the body in free expansion of it caused by the processes of heat conduction, diffusion, filtration, and phase and chemical conversion:

$$N = \beta_T\,(T - T_0) + \sum_{\psi} \beta_{x\psi}\,(\omega_\psi - \omega_{\psi 0})\,, \qquad (6)$$

where β_T is the coefficient of linear thermal expansion, $\beta_T = (\partial x/\partial T)/x$, and $\beta_{x\psi}$ is the coefficient of linear shrinkage, $\beta_{x\psi} = (\partial x/\partial \omega_\psi)/x$. We note that usually deformations due to changes in mass content of vapor and air can be neglected.

As the boundary condition for the equation of momentum transfer (5) we can take the condition of absence of stresses on the outer boundary of the region, i.e.,

$$\sigma_{ij}\big|_{v=0} = G\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right) + G_1 \frac{\partial u_j}{\partial x_i}\,\delta_{ij} - N\,(2G+3G_1)\,\delta_{ij} = 0\,, \quad i, j = 1, 2, 3\,, \tag{7}$$

where δ_{ij} is the unit tensor ($\delta_{ij} = 1$ at $i = j$ and $\delta_{ij} = 0$ at $i \neq j$).

In some cases, the solution of the problem of deformation in drying of bodies of a simple shape can be obtained analytically. For a flat plate, $0 < x_1 < H$, deformation of which is related to the inhomogeneity of the fields of concentration of the components and temperature along the x_1 axis, which is symmetric relative to the mid-plane of the plate $x_1 = H/2$, the analytical solution has the form

$$\varepsilon_{22} = \varepsilon_{33} = \frac{1}{H}\int_0^H N dx_1\,, \quad \varepsilon_{11} = \frac{1+\nu_P}{1-\nu_P}N - \frac{2\nu_P}{1-\nu_P}\varepsilon_{22}\,, \quad \varepsilon_V = (1+\varepsilon_{11})\,(1+\varepsilon_{22})\,(1+\varepsilon_{33}) - 1\,,$$

$$\varepsilon_{12} = \varepsilon_{13} = \varepsilon_{23} = 0\,, \quad \sigma_{11} = 0\,, \quad \sigma_{22} = \sigma_{33} = \frac{E_{\mathrm{el}}}{1-\nu_P}\,(\varepsilon_{22} - N)\,. \tag{8}$$

The analytical solution of the axisymmetric problem of the stress-strain state of a hollow cylinder $r_1 \leq r \leq r_0$ caused by the nonuniformity of the fields of temperature and concentration of the components and also by the effect of pressures P_0 on the inner and P on the outer cylindrical surfaces and P_z at the ends of the cylinder [8] has the form

$$u_r\,(r) = \frac{1}{r\,(1-\nu_P)}\left[(1+\nu_P)\int_{r_0}^{r} Nrdr + \frac{r^2\,(1-3\nu_P) + r_0^2\,(1+\nu_P)}{r_1^2 - r_0^2}\int_{r_0}^{r_1} Nrdr + \frac{r\nu_P P_z}{E_{\mathrm{el}}\pi\,(r_1^2 - r_0^2)}\right] +$$

$$+ \frac{1}{E_{\mathrm{el}} r\,(r_1^2 - r_0^2)}\left[r^2\,(1-\nu_P)\,(P_0 r_0^2 - P r_1^2) + (1+\nu_P)\,r_0^2 r_1^2\,(P_0 - P)\right], \tag{9}$$

$$\varepsilon_z = \frac{2}{r_1^2 - r_0^2}\int_{r_0}^{r_1} Nrdr - \frac{2\nu_P}{E_{\mathrm{el}}}\,\frac{P_0 r_0^2 - P r_1^2}{r_1^2 - r_0^2} - \frac{P_z}{E_{\mathrm{el}}\pi\,(r_1^2 - r_0^2)}\,. \tag{10}$$

Here u_r is the function of the displacement of the cylinder points in the radial direction and ε_z is the relative elongation along the axis z. At $r_0 = 0$, relations (9) and (10) are the solution of the problem of thermoconcentration elasticity for a solid cylinder of radius r_1. The components of the strain tensor ε_{rr}, $\varepsilon_{\theta\theta}$, and ε_{zz} are found from the relations

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}\,, \quad \varepsilon_{\theta\theta} = \frac{u_r}{r}\,, \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z} = \varepsilon_z\,. \tag{11}$$

Variation of the volume concentration of the substance W (energy, mass of the components, momentum) in the distinguished elementary parallelepiped with a volume $\Delta V(t) = \Delta x_1(t)\Delta x_2(t)\Delta x_3(t)$ is related to the processes of heat and mass transfer, deformation, and the effect of the sources and sinks of the substance with a power $I_W\Delta V(t)$. The amount of the substance W that flows in per time unit through the edge $\Delta x_2(t)\Delta x_3(t)$ of the parallelepiped in the plane x_1 is equal to $J_{W,x_1}\Delta x_2(t)\Delta x_3(t)$. The amount of the substance that flows out per time unit through the opposite edge in the plane $x_1 + \Delta x_1(t)$ is $J_{W,x_1+\Delta x_1}\Delta x_2(t)\Delta x_3(t)$. Expanding the function $J_{W,x_1+\Delta x_1}$ into Taylor series in powers $\Delta x_1(t)$ and restricting ourselves to the first two terms of the series, we obtain $J_{W,x_1+\Delta x_1} = J_{W,x_1} + (\partial J_{W,x_1}/\partial x)\;\Delta x_1(t)$. Expressions for the substance flows through other edges of the parallelepiped are written similarly. The amount of the substance that is accumulated by an elementary volume is equal to

$$-\left(\frac{\partial J_{W,x_1}}{\partial x_1}+\frac{\partial J_{W,x_2}}{\partial x_2}+\frac{\partial J_{W,x_3}}{\partial x_3}-I_W\right)\Delta x_1(t)\,\Delta x_2(t)\,\Delta x_3(t)=-(\nabla J_W-I_W)\,\Delta V(t)\,. \tag{12}$$

The amount of the substance W in the volume $\Delta V(t)$ at a time instant t is $W(t)\Delta V(t)$. Its variation per time unit with account for expression (4) is determined by the sum

$$\frac{\partial(W\Delta V)}{\partial t}=\Delta V\frac{\partial W}{\partial t}+W\frac{\partial\Delta V}{\partial t}=\left(\frac{\partial W}{\partial t}+\frac{W}{1+\varepsilon_V}\frac{\partial\varepsilon_V}{\partial t}\right)\Delta V\,. \tag{13}$$

Equating the right-hand sides of expressions (12) and (13), we obtain the differential equation of substance transfer for a deformable body:

$$\frac{\partial W}{\partial t}=-\nabla J_W+I_W-\frac{W}{1+\varepsilon_V}\frac{\partial\varepsilon_V}{\partial t}\,. \tag{14}$$

At $\varepsilon_V = 0$ the known Umov equation flows from (14).

The flux density J_W of the substance W in the general case has diffusion J_W^{d} and filtration J_W^{f} components: $J_W = J_W^{\text{d}}+J_W^{\text{f}}$. Mass transfer of the component ψ, when $W = U_\psi$, is carried out under the effect of the gradients of the volume concentration and temperature $J_{U\psi}^{\text{d}} = -D_\psi(\nabla U_\psi+\delta\nabla T)$. The diffusion flux of energy is composed of the heat fluxes due to the processes of heat conduction and diffusion of the mass of the components, $J_q^{\text{d}} = -\lambda\nabla T+\sum_\psi E_\psi J_{U\psi}^{\text{d}}$. The coefficient of diffusion for condensed media is determined by a formula obtained on the basis of the law of the intensity of the spectral radiation of particles [9]:

$$D=\gamma_{\text{d}}\left[\exp\left(\frac{A_{\text{d}}}{RT}\right)-1\right]^{-1}, \tag{15}$$

where A_{d} of the activation energy of molecules. When $A_{\text{d}}/(RT) >> 1$, expression (15) changes to the empirical Arrhenius formula usually used for solid bodies, and when $A_{\text{d}}/(RT) << 1$ — to the Einstein formula for liquids, for which, according to experimental data, the activation energy is much smaller than for solid bodies.

The densities of the filtration flows of liquid, vapor, and air are determined by the expressions $J_{\text{liq}}^{\text{f}} = U_{\text{liq}}w_{\text{liq}}$, $J_{\text{v}}^{\text{f}} = U_{\text{v}}w_{\text{g}}$, and $J_{\text{a}}^{\text{f}} = U_{\text{a}}w_{\text{g}}$. According to the Darcy law, the velocity of the filtration flow of the liquid w_{liq} and gaseous w_{g} phases are proportional to the gradients of their pressures ∇P_{liq} and ∇P_{g}:

$$w_\psi=-\frac{K_{\text{t}}K_\psi}{\eta_\psi}\nabla P_\psi\,,\quad \psi=\text{liq, g}\,.$$

The pressure of the vapor–gas mixture P_{g} in the body pores is expressed in terms of the volume concentrations of the components U_{v}, U_{a}, and U_{liq} and temperature T as follows. The volume fractions of liquid Ψ_{liq} and gas Ψ_{g} in the porous body are respectively equal to $\Psi_{\text{liq}} = U_{\text{liq}}/\rho_{\text{liq}}$ and $\Psi_{\text{g}} = 1-\Psi_{\text{sol}}-\Psi_{\text{liq}}$, where ρ_{liq} is the true density of the liquid. Partial densities of vapor and air in pores of the body are determined by the expressions $\rho_{\text{v}} = U_{\text{v}}/\Psi_{\text{g}}$ and $\rho_{\text{a}} = U_{\text{a}}/\Psi_{\text{g}}$. Partial pressures of vapor and air are found from the equation of state of the vapor-gas mixture. For the processes of drying it can be considered as a mixture of ideal gases. In this case, $P_{\text{v}} = \rho_{\text{v}}RT/\mu_{\text{v}}$ and $P_{\text{a}} = \rho_{\text{a}}RT/\mu_{\text{a}}$. The total pressure of the vapor–gas mixture P_{g} is equal to the sum of the partial pressures of its components:

$$P_{\text{g}}=P_{\text{v}}+P_{\text{a}}=\frac{RT}{\Psi_{\text{g}}}\left(\frac{U_{\text{v}}}{\mu_{\text{v}}}+\frac{U_{\text{a}}}{\mu_{\text{a}}}\right). \tag{16}$$

The pressure difference due to the surface-tension force arises on the surface of the contact between the vapor–air mixture and the liquid; therefore, the liquid pressure P_{liq} is presented in the form $P_{\mathrm{liq}} = P_{\mathrm{g}} + P_{\mathrm{c}}$. The pressure P_{c} is found as the mean capillary pressure of liquid in the body pores [5, 10]. In an individual cylindrical capillary of radius r, the capillary pressure, according to the Laplace formula, is $P_{\mathrm{c}} = 2\sigma(T)/r$. The volume of the liquid $dV(r)$ in the capillaries with radii from r to $r + dr$ in a unit volume of the body is proportional to the differential of the size-distribution function $f_V(r)$ of the pores (determined from the condition $\Pi = \int_{r_{\min}}^{r_{\max}} f_V(r)dr$) and the volume fraction $\theta(r)$ of the capillary occupied by the liquid, i.e., $dV(\mathrm{r}) = \theta(r)f_V(r)dr$. The mean value of the capillary pressure at the given point of the body can be presented in the form

$$P_{\mathrm{c}} = 2\sigma(T) \int_{r_{\min}}^{r_{\max}} \frac{\theta}{r} f_V dr \Big/ \int_{r_{\min}}^{r_{\max}} \theta f_V dr = \frac{2\sigma(T)}{r^*}, \tag{17}$$

which is similar to the Laplace formula.

The value of the characteristic parameter of dispersivity of the pore sizes r^* lies within the minimum $r_{\min}$ and maximum $r_{\max}$ radii of the capillary pores, i.e., $r_{\min} < r^* < r_{\max}$. The quantity $\theta = \theta(t, r)$ is determined as follows. As shown in [11], a condensate layer, whose thickness in the system equilibrium is determined by the expression $\delta = \delta^*(1 - \sqrt{1 - \varphi})$, forms on the flat surface of contact between the nonevaporating condensed body and the vapor–gas mixture. Therefore, we can approximately assume that at the time instant t the thickness of the condensate layer $\delta(t)$ in the capillary pores of radius $r \geq \delta(t)$ is the same, whereas the pores of radius $r < \delta(t)$ are completely filled with water and for them $\delta(r, t) = r$. In this case, $\theta(r, t) = 1 - [r_{\mathrm{c}}(t)/r]^2$, where $r_{\mathrm{c}}(t)$ is the radius of the liquid-free cylindrical space in the pore of radius r filled by the vapor–gas mixture $r_{\mathrm{c}}(t) = r - \delta(t)$. The value of $\delta(t)$ is determined from the condition $\rho_{\mathrm{liq}} \int_{r_{\min}}^{r_{\max}} \theta(r,t) f_V(r)dr = U_{\mathrm{liq}}$. In the equilibrium state of the system when the thickness of the condensate layer δ in the body capillaries is determined by the expression given above, the latter condition can be considered as the equation of the sorption isotherm.

The specific intensity I of the phase conversion of liquid to vapor at the boundary points of a moist porous body is determined by a formula obtained on the basis of the function of particle distribution by energies, which follows from the law of intensity of spectral radiation of particles [9]:

$$I = \gamma_{\mathrm{m}} \left\{ \bar{\delta}(2 - \bar{\delta}) \left[\exp\left(\frac{A}{RT|_{\nu=0}} \right) - 1 \right]^{-1} - \varphi_{\mathrm{m}} \left[\exp\left(\frac{A}{RT_{\mathrm{m}}} \right) - 1 \right]^{-1} \right\}. \tag{18}$$

Here $\bar{\delta} = \delta/\delta^*$ is the relative thickness of the layer of the condensed body, δ^* is the thickness of the boundary layer adjacent to the free surface of a rather massive condensed body where the evaporation process occurs, δ is the characteristic geometric parameter of the body, $0 < \delta \leq \delta^*$ (for a massive body $\delta = \delta^*$, for a thin condensed layer δ is the layer thickness), A is the activation energy for the evaporation process, $\gamma_{\mathrm{m}} = \varepsilon\rho_{\mathrm{liq}}\delta^*/4$ is the coefficient of surface evaporation, and φ_{m} is the degree of saturation of the vapor–gas mixture, $\varphi_{\mathrm{m}} = P_{\mathrm{v}}/P_{\mathrm{sat}}(T_{\mathrm{m}})$. Under the condition of rather intense supply of moisture to the boundary surface due to the processes of diffusion and filtration, we should assume that the relative thickness is $\bar{\delta} = 1$. At the end of the first period of drying, when the volume concentration of moisture in the vicinity of the boundary surface tends to an equilibrium value $U_{\mathrm{liq}}|_{\nu=0} \to U_{\mathrm{eq}}$, the humidity of the vapor–gas mixture in the body pores φ, the absolute δ and relative $\bar{\delta}$ thicknesses of the condensate layer and, thus, the intensity of evaporation decrease. It follows from the equation of energy conservation for an elementary volume adjacent to the boundary surface that at $U_{\mathrm{liq}}|_{\nu=0} \to U_{\mathrm{eq}}$ the resultant flow of vapor is

$$I = -D_{\mathrm{liq}} \left. \frac{\partial U_{\mathrm{liq}}}{\partial x} \right|_{\nu=0} . \tag{19}$$

This condition is used for determining the intensity of evaporation on the body surface in the second period of drying.

The resultant specific flow of vapor from the liquid to the gaseous phase in a unit volume of the porous body I_V is determined as follows. The area of contact between the liquid and gaseous phases in capillaries with radii from r to $r + dr$ in a unit volume of the body, where the volume fraction of the liquid phase is $\theta(t, r)$, is found from the expression $dS = 2\sqrt{1 - \theta} f(r) dr/r$. The total area of all capillaries of the unit volume is

$$S = \int_{r_{\min}}^{r_{\max}} 2 \sqrt{1 - \theta}\, f(r) dr/r .$$

According to (18) and the condition that the temperatures of the phases at each point of the body are the same, the quantity I_V can be presented in the following form [10]:

$$I_V = \gamma_V \bar{\delta}\, (2 - \bar{\delta}) \left[\exp\left(\frac{A}{RT} \right) - 1 \right]^{-1} (1 - \varphi) , \tag{20}$$

where $\gamma_V = \gamma_{\mathrm{m}} S$ is the coefficient of volumetric evaporation; $\varphi = P_{\mathrm{v}}/P_{\mathrm{sat}}(T)$.

Based on the condition of equality of the flows of evaporating and condensing particles on the free surface of the system condensed body–saturated vapor at the state of equilibrium, an analytical expression of the dependence of the pressure of saturated vapor on temperature was found in [9]:

$$P_{\mathrm{sat}} = N_{\mathrm{eq}} \sqrt{T} \left[\exp\left(\frac{A}{RT} \right) - 1 \right]^{-1} ,$$

where N_{eq} = const. Comparison of the thus-obtained values with the experimental data for water within the entire range of temperatures of its existence showed that at $A = 0.4206 \cdot 10^8$ J/mole and $N_{\mathrm{eq}} = 0.4361 \cdot 10^{10}$ kg/(m·sec^2·K$^{0.5}$) in the temperature range $0 < T < 100^{\circ}$C and $A = 0.369 \cdot 10^8$ J/mole and $N_{\mathrm{eq}} = 0.8514 \cdot 10^9$ kg/(m·sec^2·K$^{0.5}$) in the temperature range $0 < T < 394^{\circ}$C the calculated and experimental results are in agreement, with the error being close to the error of obtaining experimental data.

As a result of substitution of the above-given expressions for the coefficient of diffusion, the intensity of volumetric evaporation, and the dependences of velocities and flux-densities of the mass of the components on their volume concentration into the equation of substance transfer (14), a system of equations that describes the interrelated processes of heat and mass transfer, phase conversions, and shrinkage in dehydration of colloidal capillary-porous bodies is constructed:

$$\frac{\partial U_{\mathrm{liq}}}{\partial t} + \nabla (w_{\mathrm{liq}} U_{\mathrm{liq}}) = \nabla [D_{\mathrm{liq}} (\nabla U_{\mathrm{liq}} + \delta_{\mathrm{liq}} \nabla T)] - \gamma_V \left[\exp\left(\frac{A}{RT} \right) - 1 \right]^{-1} (1 - \varphi) - \frac{U_{\mathrm{liq}}}{1 + \varepsilon_V} \frac{\partial \varepsilon_V}{\partial t} , \tag{21}$$

$$\frac{\partial U_{\mathrm{v}}}{\partial t} + \nabla (w_{\mathrm{v}} U_{\mathrm{v}}) = \nabla [D_{\mathrm{v}} (\nabla U_{\mathrm{v}} + \delta_{\mathrm{v}} \nabla T)] + \gamma_V \left[\exp\left(\frac{A}{RT} \right) - 1 \right]^{-1} (1 - \varphi) - \frac{U_{\mathrm{v}}}{1 + \varepsilon_V} \frac{\partial \varepsilon_V}{\partial t} , \tag{22}$$

$$\frac{\partial U_{\mathrm{a}}}{\partial t} + \nabla (w_{\mathrm{a}} U_{\mathrm{a}}) = \nabla [D_{\mathrm{a}} (\nabla U_{\mathrm{a}} + \delta_{\mathrm{a}} \nabla T)] - \frac{U_{\mathrm{a}}}{1 + \varepsilon_V} \frac{\partial \varepsilon_V}{\partial t} , \tag{23}$$

$$c_{\mathrm{ef}}\left(\frac{\partial T}{\partial t}+w_{\mathrm{ef}}\nabla T\right)=\nabla\,(\lambda_{\mathrm{ef}}\nabla T)-\sum\,c_{\psi}D_{\psi}\,(\nabla U_{\psi}+\delta\nabla T)\,\nabla T-L\gamma_{V}\left[\exp\left(\frac{A}{RT}\right)-1\right]^{-1}(1-\varphi)\,, \tag{24}$$

where

$$\varphi=U_{\mathrm{v}}/\Psi_{\mathrm{g}}P_{\mathrm{sat}}\,(T)\,;\quad c_{\mathrm{ef}}=c_{\mathrm{sol}}U_{\mathrm{sol}}+c_{\mathrm{liq}}U_{\mathrm{liq}}+c_{\mathrm{v}}U_{\mathrm{v}}+c_{\mathrm{a}}U_{\mathrm{a}}\,;$$

$$\lambda_{\mathrm{ef}}=\lambda_{\mathrm{sol}}U_{\mathrm{sol}}/\rho_{\mathrm{sol}}+\lambda_{\mathrm{liq}}U_{\mathrm{liq}}/\rho_{\mathrm{liq}}+\lambda_{\mathrm{v}}U_{\mathrm{v}}/\rho_{\mathrm{v}}+\lambda_{\mathrm{a}}U_{\mathrm{a}}/\rho_{\mathrm{a}}\,;$$

$$w_{\mathrm{ef}}=[w_{\mathrm{liq}}U_{\mathrm{liq}}+w_{\mathrm{g}}\,(U_{\mathrm{v}}+U_{\mathrm{a}})]/c_{\mathrm{ef}}\,.$$

The specific heat of water evaporation L inside a porous material depends on its moisture content and is composed of the specific heat of evaporation of free liquid and the specific energy of moisture desorption. Equation (24) is obtained by combining the energy equation in the form (14) and the equations of transfer of the substance mass (21)–(23). Usually, the second term on the right-hand side of Eq. (24), which is caused by energy transfer due to diffusion, can be neglected [1].

For the system of equations (21)–(24) the boundary conditions of heat and mass transfer provided that the parameters of the drying agent are specified are formulated as follows:

$$\lambda\left.\frac{\partial T}{\partial \nu}\right|_{\nu=0}=\alpha\,(T_{\mathrm{m}}-T|_{\nu=0})-LI\,, \tag{25}$$

$$D_{\mathrm{liq}}\left(\lambda\left.\frac{\partial U_{\mathrm{liq}}}{\partial \nu}\right|_{\nu=0}+\;\delta_{\mathrm{liq}}\left.\frac{\partial T}{\partial \nu}\right|_{\nu=0}\right)+J^{\mathrm{f}}_{\mathrm{liq}}|_{\nu=0}=I\,, \tag{26}$$

$$-D_{\mathrm{v}}\left(\left.\frac{\partial U_{\mathrm{v}}}{\partial \nu}\right|_{\nu=0}+\;\delta_{\mathrm{v}}\left.\frac{\partial T}{\partial \nu}\right|_{\nu=0}\right)+J^{\mathrm{f}}_{\mathrm{v}}|_{\nu=0}=\gamma_{\mathrm{v,m}}\,(U_{\mathrm{v}}|_{\nu=0}-\rho_{\mathrm{v,m}}\psi_{\mathrm{v}})+\gamma_{\mathrm{v,m}}\delta_{\mathrm{v,m}}\psi_{\mathrm{v}}\,(T|_{\nu=0}-T_{\mathrm{m}})\,. \tag{27}$$

We note that expression (27) satisfies the conditions of the body entry into the equilibrium state when $t\to\infty$: $\rho_{\mathrm{v}}=\rho_{\mathrm{v,m}}$, $T=T_{\mathrm{m}}$, and $\partial U/\partial\nu=\partial T/\partial\nu=0$ [10].

The volume concentration of air on the outer boundary is determined from the condition that the pressure of the vapor–gas mixture at the inner points of the body in the vicinity of this boundary be equal to the pressure of the outer medium, i.e.,

$$P_{\mathrm{g}}|_{\nu=0}=P_{\mathrm{v}}|_{\nu=0}+P_{\mathrm{a}}|_{\nu=0}=P_{\mathrm{m}}=P_{\mathrm{v,m}}+P_{\mathrm{a,m}}\,.$$

In this case, we write the condition of mass transfer for air as

$$U_{\mathrm{a}}|_{\nu=0}=\frac{P_{\mathrm{m}}\Psi_{\mathrm{g}}\mu_{\mathrm{a}}}{RT|_{\nu=0}}-U_{\mathrm{v}}|_{\nu=0}\,\frac{\mu_{\mathrm{a}}}{\mu_{\mathrm{v}}}\,. \tag{28}$$

In the first period of drying, when the volume concentration of moisture on the outer boundary of the body $U_{\mathrm{liq}}|_{\nu=0}>0$, the specific volumetric rate of evaporation I is determined by formula (18) at $\bar{\delta}=1$; in the second period of drying, when $U_{\mathrm{liq}}|_{\nu=0}=0$, it is determined from condition (19).

The formulated mathematical model of diffusion-filtration heat and mass transfer in the processes of dehydration of porous materials can undergo substantial simplification in its application to description of the specific processes of drying. Filtration transfer [1, 4] strongly influences the dynamics of drying in intense heating of the body when evaporation of liquid occurs at a temperature $T\geq 100^{\mathrm{o}}$C or due to the difference in pressures between media separated by a porous wall. For the case where the effect of filtration on the processes of heat and mass transfer can be neglected, the mathematical model of the process of drying, based on the system of equations (21)–(24), can be presented in the following form:

$$c_{\text{ef}} \frac{\partial T}{\partial t} = \nabla (\lambda_{\text{ef}} \nabla T) - L\gamma_V \left[\exp \left(\frac{A}{RT} \right) - 1 \right]^{-1} (1 - \varphi) , \tag{29}$$

$$\frac{\partial U_{\text{liq}}}{\partial t} = \nabla \left[D_{\text{liq}} \left(\nabla U_{\text{liq}} + \delta_{\text{liq}} \nabla T \right) \right] - \gamma_V \left[\exp \left(\frac{A}{RT} \right) - 1 \right]^{-1} (1 - \varphi) - \frac{U_{\text{liq}}}{1 + \varepsilon_V} \frac{\partial \varepsilon_V}{\partial t} , \tag{30}$$

$$\frac{\partial U_{\text{v}}}{\partial t} = \nabla \left[D_{\text{v}} \left(\nabla U_{\text{v}} + \delta_{\text{v}} \nabla T \right) \right] + \gamma_V \left[\exp \left(\frac{A}{RT} \right) - 1 \right]^{-1} (1 - \varphi) - \frac{U_{\text{v}}}{1 + \varepsilon_V} \frac{\partial \varepsilon_V}{\partial t} . \tag{31}$$

In the absence of filtration, the pressure of the vapor–gas mixture is constant and is equal to the pressure of the outer medium P_{m}; therefore, the partial pressure of air P_{a} is determined by the expression $P_{\text{a}} = P_{\text{m}} - P_{\text{v}}$. For this case, in the boundary conditions (26) and (27) we must assume $J^{\text{f}}_{\text{liq}} = J^{\text{f}}_{\text{v}} = 0$.

The systems of equations (21)–(24) and (29)–(31) that describe the interrelated processes of heat and mass transfer and phase conversions with and without account for filtration of the bound substance in drying are substantially nonlinear. They can be realized on the basis of numerical methods.

Numerical Method of Calculation of the Drying Process. Equations of transfer (21)–(24) involve convective terms, which determine the intensity of the processes of filtration of the liquid and vapor–gas mixture. Such partial-differential equations can be solved on the basis of the explicit three-layer recalculation difference scheme suggested in [7]. The conditions of stability do not impose limitations on the spatial steps of the difference grid, and the limitations on the time steps can be reduced to the dependence on the velocity of convective (filtration) motion. By virtue of the fact that the right-hand sides of Eqs. (21)–(23) have terms with the function of the volumetric strain ε_{V}, the determination of which in the physical sense is an independent problem of thermoconcentration strain [6], in numerical realization of these equations it is expedient to use the procedure of splitting of the algorithm by physical factors [12]. In this case, three differential equations correspond to each differential equation of mass transfer and two differential equations correspond to the equation of heat transfer.

In the Cartesian coordinates x_1, x_2, x_3 on the difference grid $x^n_{i,m_i+1} = x^n_{i,m_i} + h^n_{i,m_i+1}$ ($m_i = 0, 1, ..., M_i$; $i = 1, 2, 3$), $t_n = nl$ ($n = 0, 1, ..., l > 0$), differential approximation of Eqs. (21)–(24) where, for the sake of simplicity, the terms related to thermal diffusion are omitted, can be presented in the following form:

$$\delta_t \bar{U}_{\text{liq}} = - \sum_{i=1}^{3} \delta_i \left(w_{\text{liq}i} U_{\text{liq}} \right) , \tag{32}$$

$$(1 + \Omega_{\text{liq}}) \, \delta_i \tilde{U}^n_{\text{liq}} - \Omega_{\text{liq}} \delta_t U^{n-1}_{\text{liq}} = - \sum_{i=1}^{3} \delta_i \left(w_{\text{liq}i} \bar{U}_{\text{liq}} \right) + \left\{ \sum_{i=1}^{3} \left[\delta_i \left(D_{\text{liq}i} \delta_i \bar{U}_{\text{liq}} \right) \right] - \gamma_V \left[\exp \left(\frac{A}{RT} \right) - 1 \right]^{-1} (1 - \varphi) \right\} , \tag{33}$$

$$\delta_t \tilde{U}_{\text{liq}} = \frac{\tilde{U}_{\text{liq}}}{1 + \varepsilon_V} \, \delta_t \varepsilon_V , \tag{34}$$

$$\delta_t \bar{U}_{\text{v}} = - \sum_{i=1}^{3} \delta_i \left(w_{\text{g}i} U_{\text{v}} \right) , \tag{35}$$

$$(1+\Omega_{\text{v}})\,\delta_t\tilde{U}_{\text{v}}^{n} - \Omega_{\text{v}}\,\delta_t U_{\text{v}}^{n-1} = -\sum_{i=1}^{3} \delta_i\,(w_{\text{g}i}\overline{U}_{\text{v}}) + \left\{\sum_{i=1}^{3} [\delta_i\,(D_{\text{v}}\delta_i\overline{U}_{\text{v}})] + \gamma_V\left[\exp\left(\frac{A}{RT}\right) - 1\right]^{-1} (1-\varphi)\right\}, \tag{36}$$

$$\delta_t\tilde{U}_{\text{v}} = \frac{\tilde{U}_{\text{v}}}{1+\varepsilon_V}\,\delta_t\varepsilon_V\,, \tag{37}$$

$$\delta_t\overline{U}_{\text{a}} = -\sum_{i=1}^{3} \delta_i\,(w_{\text{g}i}U_{\text{a}})\,, \tag{38}$$

$$(1+\Omega_{\text{a}})\,\delta_t\tilde{U}_{\text{a}}^{n} - \Omega_{\text{a}}\delta_t U_{\text{a}} = -\sum_{i=1}^{3} \delta_i\,(w_{\text{g}i}\overline{U}_{\text{a}}) + \sum_{i=1}^{3} [\delta_i\,(D_{\text{a}}\delta_i U_{\text{a}})]\,, \tag{39}$$

$$\delta_t\tilde{U}_{\text{a}} = \frac{\tilde{U}_{\text{a}}}{1+\varepsilon_V}\,\delta_t\varepsilon_V\,, \tag{40}$$

$$\delta_t\overline{T} = -\sum_{i=1}^{3} w_{\text{ef}i}\delta_i T\,, \tag{41}$$

$$(1+\Omega_T)\,\delta_t T^{n} - \Omega_T\delta_t T^{n-1} = -\sum_{i=1}^{3} w_{\text{ef}i}\delta_i\overline{T} + \frac{1}{c_{\text{ef}}}\left\{\sum_{i=1}^{3} [\delta_i\,(\lambda_{\text{ef}}\delta_i\overline{T})] - L\gamma_V\left[\exp\left(\frac{A}{RT}\right) - 1\right]^{-1} (1-\varphi)\right\}. \tag{42}$$

In the differential equations (32)–(42), the grid functions $\Phi^{n}_{m_1m_2m_3}$, $\overline{\Phi}^{n+1}_{m_1m_2m_3}$, and $\tilde{\Phi}^{n+1}_{m_1m_2m_3}$ ($\Phi = T$, U_{liq}, U_{v}, U_{a}, ε_V) for the nodal point $(x_{1,m_1}, x_{2,m_2}, x_{3,m_3}, t_n)$ are written, for the sake of simplicity, without the sub- and superscripts, i.e., $\Phi = \Phi^{n}_{m_1m_2m_3}$, $\overline{\Phi} = \overline{\Phi}^{n+1}_{m_1m_2m_3}$, and $\tilde{\Phi} = \tilde{\Phi}^{n+1}_{m_1m_2m_3}$; Ω is the weight factor that allows one to eliminate the limitation to the time step, which is due to the diffusion terms in Eqs. (21)–(24), $\Omega \ge 0$;

$$\delta_t\Phi = \frac{\Phi^{n+1}_{m_1m_2m_3} - \Phi^{n}_{m_1m_2m_3}}{l}\,;\quad \delta_t\Phi^{n-1} = \frac{\Phi^{n}_{m_1m_2m_3} - \Phi^{n-1}_{m_1m_2m_3}}{l}\,;\quad \delta_1\Phi = \frac{\Phi^{n}_{m_1+1,m_2m_3} - \Phi^{n}_{m_1-1,m_2m_3}}{x^{n}_{m_1+1} - x^{n}_{m_1-1}}\,;$$

$$\delta_1\,(D\delta_1\Phi) = \left[(D_{m_1+1,m_2m_3} + D_{m_1m_2m_3})\,\frac{\Phi^{n}_{m_1+1,m_2m_3} - \Phi^{n}_{m_1m_2m_3}}{x^{n}_{m_1+1} - x^{n}_{m_1}} - (D_{m_1m_2m_3} + \right.$$

$$\left. + D_{m_1-1,m_2m_3})\,\frac{\Phi^{n}_{m_1m_2m_3} - \Phi^{n}_{m_1-1,m_2m_3}}{x^{n}_{m_1} - x^{n}_{m_1-1}}\right] \Big/ \left(x^{n}_{m_1+1} - x^{n}_{m_1-1}\right).$$

The error of approximation of differential equations (21)–(24) by the difference scheme (32)–(42) has the order $l + h^2_{m_1} + h^2_{m_2} + h^2_{m_3}$. The necessary conditions of stability of the solution of the system of differential equations (32)–(42) to the solution of differential equations (21)–(24), which are determined on the basis of the method of conventional assignment of some unknown functions of the system [6], can be written as follows:

$$l_{\psi} \le \min \left[\left(\sum_{i=1}^{3} \frac{w_{\psi i}}{h_i} \right)^{-1} ;\ \ (1 + 2\Omega_{\psi}) \left(2B_{\psi} \sum_{i=1}^{3} \frac{1}{h_i^2} \right)^{-1} \right] . \tag{43}$$

At ψ = liq and $B_{\text{liq}} = D_{\text{liq}}$, condition (43) provides stability of Eqs. (32)–(34), at ψ = v and $B_{\text{v}} = D_{\text{v}}$ — that of Eqs. (35)–(37), at ψ = a and $B_{\text{a}} = D_{\text{a}}$ — that of Eqs. (38)–(40), and at $w_T = w_{\text{ef}}$ and $B_T = \lambda_{\text{ef}}/c_{\text{ef}}$ — that of Eqs. (41) and (42). The time step l is found from the condition

$$l \le \min \left\{ l_T;\ l_{\text{liq}};\ l_{\text{v}};\ l_{\text{a}} \right\} . \tag{44}$$

In the general case of a multidimensional porous body, the function of volumetric strain ε^{n+1} and the coordinates of the nodal points on the time layer $n + 1$ are found from the solution of the system of equations (5) with conditions (6) and (7) by the time-dependent technique based on the three-layer difference scheme [7]. In this case, differential approximation of Eq. (5) on the orthogonal difference grid has the form

$$(1 + \Omega_{\text{el}})\, \delta_t u_i^n - \Omega_{\text{el}} \delta_t u_i^{n-1} = \sum_{j=1}^{3} \partial_j \,(G \partial_j u_i^n) + \sum_{j=1}^{3} \partial_i \,(\lambda \partial_j u_j^n) + \sum_{j=1}^{3} \partial_j \,(G \delta_j u_j^n) - \partial_i \,[N\,(2G + 3\lambda)] \,, \tag{45}$$

where $\Omega_{\text{el}} \ge 0$ and i, j = 1, 2, 3. The necessary conditions of stability of (45) found on the basis of the method of conventional assignment of some unknown functions of the system are [7]

$$\frac{2l_{\text{el}}G}{1 + 2\Omega_{\text{el}}} \left[\frac{1}{h_1^2} + \frac{1}{h_2^2} + \frac{1}{h_3^2} + \left(1 + \frac{G_1}{G} \right) \frac{1}{h_i^2} \right] \le 1 \,, \quad \frac{h_i}{2G} \frac{\partial G}{\partial x_i} \le 1 \,, \quad \frac{h_i}{2\,(2G + G_1)} \frac{\partial\,(2G + G_1)}{\partial x_i} \le 1 \,. \tag{46}$$

For a partial case of porous plate drying $(0 < x_1 < H)$, the volumetric strain ε and coordinates of the nodal points of the difference grid on the $(n + 1)$th layer can be found based on the analytical solution (8). In the process of dehydration, the coordinates of the nodal points x_m^n (m = 0, 1, ..., M, $x_M^0 = H$) undergo changes due to shrinkage; however, in each elementary layer $x_{m+1}^n - x_m^n$ the mass of the solid (skeleton) remains constant, i.e.,

$$U_T^n \,(x_{m+1}^n - x_m^n)\, S^n = U_T^0 \,(x_{m+1}^0 - x_m^0)\, S^0 = g_{Tm} \,,$$

where S^n is the plate surface area in the plan (in the plane $x_2 0 x_3$), $S^n = S^0[1 - (\varepsilon')^n]^2$. The components of the strain tensor $\varepsilon_{22}^{n+1} = \varepsilon_{33}^{n+1} = (\varepsilon')^{n+1}$ on the $(n + 1)$th time layer are determined by the first expression of (8) with the function N^{n+1} being found from formula (6), where $T = T^{n+1}$, $W = W^{n+1} = \tilde{U}_m^{n+1}(x_{m+1}^n - x_{m-1}^n)S^n/[g_T^0(x_{m+1}^0 - x_{m-1}^0)S^0]$, and for calculation of the integral the trapezoid formula can be used. According to the third formula in (8), variation of the grid step along the x_1 axis is

$$h_{m+1}^{n+1} = h_{m+1}^n \left\{ 1 - \left[\frac{1 + \nu_{\text{v}}}{1 - \nu_{\text{v}}} \left(\frac{N_{m+1}^{n+1} + N_m^{n+1}}{2} - \frac{N_{m+1}^n + N_m^n}{2} \right) - \frac{2\nu_{\text{v}}}{1 - \nu_{\text{v}}} \left(\varepsilon'^{\,n+1} - \varepsilon'^{\,n} \right) \right] \right\} . \tag{47}$$

The derivative $\delta_t \varepsilon$ at the nodal points of the plate is determined as follows:

$$\delta_t \varepsilon = \frac{S^{n+1}\,(h_{m+1}^{n+1} + h_m^{n+1}) - S^n\,(h_{m+1}^n - h_m^n)}{lS^n\,(h_{m+1}^n + h_m^n)} \,. \tag{48}$$

The grid function of the volume fraction of the porous body skeleton on the $(n + 1)$th layer is found by a differential equation that approximates (4):

TABLE 1. Temperature and Duration of Treatment at Different Stages of Drying

Stages of thermal treatment	Temperature of treatment, $^\mathrm{o}$C	Duration of treatment, min
1	85—90	25—30
2	75—80	25—30
3	68—72	30—45
4	60—67	35—50
5	45—50	50—60

$$\partial_t \psi = -\frac{\psi_m}{1+\varepsilon_V}\,\partial_t \varepsilon_V \,. \tag{49}$$

Two methods can be used for determination of the values of the unknown functions T, U_liq, U_v, and U_a at the boundary nodal points of the region on the $(n+1)$th time layer: in the first method, difference approximations of the boundary conditions of heat and mass transfer (25)–(28) are used; in the second method — differential equations approximating initial differential equations of transfer (21)–(24) that are similar in form to (32)–(42) and are constructed with account for conditions (25)–(28). Approximation by the first method of the boundary condition, e.g., (25), for the nodal point $(x_{1,0},\ x_{2,m_2},\ x_{3,m_3})$ lying on the boundary $x_1 = 0$ has the form

$$\lambda\,\frac{T^{n+1}_{0m_2m_3} - T^{n+1}_{1m_2m_3}}{h_1} = \alpha\left(T_\mathrm{m} - T^{n+1}_{0m_2m_3}\right) - LI \,. \tag{50}$$

The system of differential equations (32)–(42) can be used to solve Eqs. (29)–(31), which describe transfer processes without regard for filtration of the vapor–gas mixture and liquid. In this case, Eqs. (38)–(40) are discarded, and Eqs. (32), (35), and (41) change over to the identities $\overline{\Phi}^{n+1}_{m_1m_2m_3} = \Phi^{n}_{m_1m_2m_3}$ ($\Phi = T,\ U_\mathrm{liq},\ U_\mathrm{v}$); in Eqs. (33), (36), and (42) and in the condition of stability (43) the terms involving the components of the velocity vector w_ψ (ψ = liq, g) are omitted.

The Energy- and Resource-Saving Method of Drying of Thermolabile Materials. Several methods of convective drying of thermolabile materials that utilize stepwise variation of the drying-agent temperature are known. Thus, for example, Borovskii et al. [13] describe a method of drying of fruit pomace according to which dehydration is performed in two stages. In the first stage, blowing by hot air is performed for 50–60 min (air temperature 98–100$^\mathrm{o}$C) and in the second stage for 25–30 min (air temperature 50–60$^\mathrm{o}$C). In this case, the moisture content of air is 10–15 g of water per kilogram of dry air. However, results of the experimental and theoretical studies indicate that the time of the drying process greatly depends on many factors: initial humidity and temperature of the material, its geometric and thermophysical parameters, temperature, humidity, and velocity of motion of the drying agent. Therefore, this method of drying with two-stage variation of the temperature of hot air can be used only for specific materials and a certain range of values of the indicated parameters and it is not optimal as regards energy consumption.

A method of drying of split aubergines by thermal treatment of the material by an air flow in five stages, the characteristics of which are given in Table 1, is suggested in [14]. The air velocity at all stages is 0.5–1.5 m/sec. This method is also not universal. It can be used for drying a given material with a specific size, initial humidity, etc. In stepwise methods of drying, the product temperature could exceed the maximum permissible one, thus leading to deterioration of the product quality and also the occurrence of the process of drying at a temperature that is lower than the maximum permissible one, which causes an increase in the period of drying and, consequently, energy losses.

In what follows, we formulate a new resource- and energy-saving method of drying of thermolabile materials, which can be used in the food, construction, medicine, and other branches of industry. It includes two stages [15]. In the first stage, dehydration is performed by the drying agent the temperature T_m of which is higher than the maximum permissible T^* for this material. In the general case, it can be assigned as a function of time, $T_\mathrm{m} = T_\mathrm{m}(t)$, which changes from a value close to the maximum one for this drying facility to the maximum permissible one for this material. In this case, the parameters of the drying agent must be selected such that the temperature of the wet-bulb thermometer T_w be lower than the maximum permissible temperature. Then the time of completion of the first stage exceeds the time of the first period of drying till the termination of which the volume concentration of moisture on the outer boundary of the body is $U_{\text{liq}}\big|_{\nu=0} = 0$ and remains such till the completion of drying. The first stage of drying terminates when the temperature on the outer boundary of the body reaches a value T^*.

In the second stage of drying, a constant temperature equal to the maximum permissible one T^* is maintained on the outer boundary of the body. This condition can be satisfied by varying the temperature of the drying agent, its moisture content, or the velocity of motion or simultaneous variation of the temperature, moisture content, and velocity of motion of the heat carrier.

In order to realize the suggested method of drying one must know the graph of variation of the drying-agent temperature or other parameters that affect the intensity of heat exchange between the agent and the moist porous body. It should be noted that this graph must be determined depending on the type of the material, its geometric and thermophysical characteristics, the initial values of temperature and moisture content, and the velocity, temperature and humidity of the drying agent and some other parameters. Experimental determination of such graphs poses considerable difficulty. Therefore, a rational way of obtaining them is by using the above-formulated method of mathematical simulation of the dynamics of the processes of heat and mass transfer and phase conversions in drying of thermolabile porous materials.

Mathematical simulation of the dynamics of the processes of heat and mass transfer at the first stage of drying of thermolabile materials is performed directly on the basis of the mathematical model and the method of calculation given earlier. In order to realize the second stage of drying, one must determine such a function of variation of the parameters of the drying agent with time that could provide maintenance of the maximum permissible temperature on the outer boundary. If the condition $T\big|_{\nu=0} = T^*$ is satisfied by variation of only the temperature of the drying agent T_{m}, then this temperature can be found from the expression

$$T_{\text{m}} = T\big|_{\nu=0} + \left[\lambda \frac{\partial T}{\partial \nu}\bigg|_{\nu=0} + LI \right] \Big/ \alpha \,, \tag{51}$$

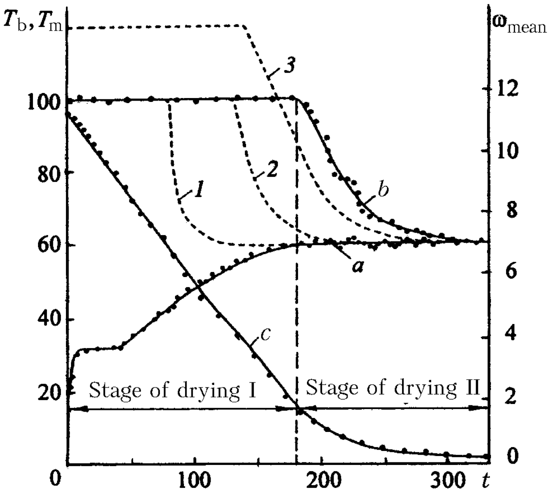
which follows from the boundary condition (25). In this case, the rate of liquid evaporation I is determined by conditions (19) and $U_{\text{liq}}\big|_{\nu=0} = 0$. In accordance with this, in the computational algorithm the difference approximation of Eq. (51), which can be written as

$$T_{\text{m}} = T^* + \frac{\lambda}{\alpha h_1}\left(T^* - T^{n+1}_{0m_2m_3}\right) + \frac{L}{\alpha h_1} U_{\text{liq},0m_2m_3} \tag{52}$$

is used instead of the difference equation (50) for the surface $x_1 = 0$.

The suggested method of drying is realized as follows. Before the process of dehydration, the thickness of the layer of the material subject to drying is determined. By the reference data and measurement of the thermophysical characteristics of the material, the maximum permissible temperature, which can be a function of the material humidity, and initial values of its humidity and temperature are found. The flow rate and temperature of the drying agent supplied to the drying chamber during the first stage of drying are specified. Using the suggested numerical method of calculation of the dynamics of drying, we find the graph of variation of the temperature of the drying agent. This graph or the corresponding table constructed on its basis in combination with the above-given initial parameters of the process completely determine the mode of drying of the thermolabile material.

Based on the formulated mathematical model and the numerical method of calculation, we developed a program complex for computer modeling of heat and mass transfer, phase conversions, and deformation in drying of colloidal capillary-porous bodies. Using this complex, we calculated the dynamics and the kinetics of drying of a layer of table beet ($0 < x_i < H$) symmetrically blown by hot air at the following initial parameters [1, 16]: $U_{\text{liq},0} = 904$ kg/m^3, $T_0 = 293$ K, $T_{\text{m}}^0 = 373$ K, $T^* = 60^{\text{o}}$C, $P_{\text{m}} = 2\cdot10^3$ Pa, $\Pi = 0.924$; $H = 12.5$ mm, 9 mm, 6 mm; $\alpha = 37$ W/(m^2·K), $\gamma_{\text{d}} = 0.5\cdot10^{-9}$ m^2/sec, $\lambda_{\text{sol}} = 0.13$ W/(m·K), $c_{\text{sol}} = 1465$ J/(kg·K), $\rho_{\text{sol}} = 1050$ kg/m^3, $d_{\text{m}} = 9.6$ g/kg of dry substance, $w_{\text{m}} = 1$ m/sec, $\beta_x = 0.037$, $\gamma_{V0} = 0.1177\cdot10^4$ kg/(m^2·sec), $\gamma_{\text{m}} = 0.118\cdot10^5$ kg/(m^2·sec), and $A = A_{\text{d}} = 0.4206\cdot10^8$ J/kmole. As a result of the calculations, we obtained graphs of the variation of the drying-agent temperature T_{m}, the temperature on the outer boundary of the layer T_{b}, and the mean volume concentration of liquid U_{liq}.



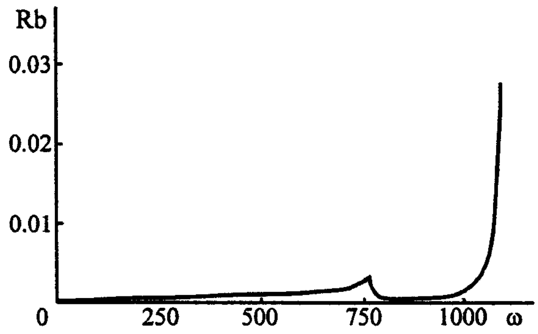


Fig. 1. Measurement of the temperature on the outer surface of the beet layer T_b (a), mean moisture content ω_{mean} (b) (H = 12.4 mm, T_m^0 = 100°C), and temperature of the drying agent T_m (c) in time: 1) H = 6 mm, T_m^0 = 100°C; 2) 9 and 100; 3) 12.5 and 120 (points — experiment, curves — calculated curves). T_b, T_m^0, °C; ω_{mean}, kg/kg; t, min.

Fig. 2. Dependence of the Rebinder number Rb on the moisture content ω in drying of a layer of table beet, ω, %.

In Fig. 1, the solid curves give the graphs of variation of temperatures T_m and T_b and mean moisture content ω_{mean} in drying of a plate of thickness H = 12.5 mm by a drying agent with an initial temperature of T_m^0 = 100°C. According to this graph, the duration of the first stage of drying is 180 min. The dashed curves in the figure show the graphs of variation of the drying-agent temperature for different values of the initial temperature of the drying-agent and plate thickness.

The graphs of variation of the drying-agent temperature that are given in the figure indicate their great dependence on the initial parameters of drying. The results of the experimental study of the mean moisture content of the body, its temperature, and the drying-agent temperature, which are given in the figure by points, and the time of the drying process are in good agreement with the calculated data. The deviation of temperature on the surface of the beet layer at the second stage of drying from the maximum permissible temperature T^* did not exceed 2°C; thus, the product quality corresponds to the standard requirements. The differences between the calculated and measured values of the drying-agent temperature do not exceed 3°C. This indicates that the process of drying of the considered layer of beet can be controlled by the dependence $T_m = T(t)$ shown in Fig. 1. Since the graphs of variation of the temperature of the drying agent or other parameters that affect the rate of heat exchange with a capillary-porous body are one-parameter, continuous, and monotonic dependences, there is the possibility of rather simply automating the monitoring of the drying process.

The Rebinder number [1]

$$\mathrm{Rb} = \frac{c_{ef}}{L}\frac{d\bar{T}}{d\bar{\omega}},$$

which determines the ratio of the amount of heat spent toward heating of the body to the amount of heat spent toward evaporation of moisture, in an infinitely small period of time, can serve as the optimization criterion of the process of drying.

Figure 2 shows the dependence of the Rebinder number Rb on the moisture content ω of the layer of table beet of thickness H = 12.5 mm at an initial temperature of the drying agent of T_m^0 = 100°C and the maximum permissible temperature T^* = 60°C; the beet layer was dried by the above-described method. At the initial stage of the drying process, when intense warming of the layer takes place, the number Rb is rather high. As the material temperature approaches the temperature of the wet-bulb thermometer, the number Rb decreases and at the first stage of drying it remains virtually zero. At the beginning of the second period of drying the temperature of the wet body increases and the number Rb becomes higher. As the boundary surface of the material approaches the maximum permissible temperature T^*, the second stage of the suggested method of drying begins; at this stage, the temperature on the body surface stays constant and equal to T^*. From this moment the number Rb decreases monotonically, approaching zero as the layer is heated to the temperature T^*. The fact that during the first and second periods of drying the number Rb does not exceed 0.0035 indicates that virtually all heat from the drying agent is spent to ward evaporation of moisture and consequently the process of drying occurs with minimum losses of heat [16].

Results of physical and numerical experiments indicate that energy consumption and duration of the process of convective drying of colloidal capillary-porous materials usually decrease as the time-mean temperature of the drying agent increases. We can show that for specified values of the maximum temperature of the drying agent T_{m} and the maximum permissible temperature T^* of the material subject to drying, the formulated method of drying, other conditions being equal, gives the possibility of providing a minimum period of drying and minimum energy losses. Although the excess of the temperature of the heat carrier relative to the maximum permissible temperature at the second stage of drying leads to acceleration of the process of material dehydration, it causes deterioration of the quality of the finished product and, therefore, is not expedient. The decrease in temperature at the second stage of drying leads to an increase in the duration of the dehydration process and in energy consumption, Thus, realization of the suggested method of drying allows energy and reource savings.

CONCLUSIONS

1. A mathematical model has been formulated, and a numerical method of calculation of the processes of heat and mass transfer, phase conversions, and shrinkage in drying of thermolabile, colloidal, capillary-porous bodies has been suggested. The method allows one to conduct multiversion variant calculations of the fields of temperature, volume concentration, and pressure in each component of the bound material in order to select rational regimes of drying.

2. Based on the results of numerical simulation, the effect of different outer factors on the dynamics and kinetics of the process of dehydration of porous bodies is analyzed and a new energy- and resource-saving method of drying of thermolabile materials is developed. According to this method, dehydration is performed in two stages. In the first stage, the body temperature increases gradually up to the maximum permissible value. In the second stage, the temperature on the body surface is maintained equal to the maximum permissible one by varying the temperature of the drying agent, the velocity of its motion or by simultaneous variation of the temperature, humidity, and velocity of motion of the agent. This method allows one to maximally decrease the time of drying, thus decreasing the consumption of energy for drying.

NOTATION

A, activation energy of molecules; c, specific isobaric heat capacity; d_{m}, moisture content of the drying agent; D, coefficient of diffusion; E_{el}, elasticity modulus; E_ψ, specific enthalpy of the ψth component; f_V, differential distribution function of pores over the volume; h and l, size of steps of the difference grid along the space coordinate and time; H, plate thickness; I, intensity of phase conversion of liquid to vapor on the boundary surface of the body; I_V, rate of evaporation in a unit volume of the porous body; I_W, power of the sources of substance W; J, density of liquid, vapor, and air flows; K_{t}, total permeability of the medium; K_ψ, relative permeability of the phase; L, specific heat of phase conversion of liquid to vapor; N, thermal concentration function; P, pressure; P_{c}, capillary pressure; r, radius; r^*, characteristic parameter of dispersion of pore sizes; R, universal gas constant; Rb, Rebinder number; S, area; t, time; T, temperature; T_{w}, temperature of the wet-bulb thermometer; T^*, maximum permissible temperature for the given material; u_i, component of the displacement vector, $i = 1, 2, 3$; U, volume concentration of the bound substance; V, volume; w, velocity of filtration motion of the phase; x, z, coordinate axes; α, heat-transfer coefficient; β_T, coefficient of linear thermal expansion; β_ψ, coefficient of linear shrinkage; γ, coefficient of evaporation; $\gamma_{\text{v,sur}}$, coefficient of outer vapor transfer; γ_{d}, constant parameter in the formula for the coefficient of diffusion; δ_ψ, relative coefficient of thermodiffusion, ψ = liq, v, a; δ, thickness of the condensate layer; $\bar{\delta}$, relative thickness of the layer of the condensed body; ε, average coefficient of the intensity of radiation of liquid particles; ε_{ij}, component of the strain tensor, i, j = 1, 2, 3; ε_V, relative volumetric strain; η, dynamic coefficient of viscosity; $\theta(r)$, fraction of the volume of capillaries of radius r filled by liquid; λ, thermal conductivity; μ, molecular mass of substance; ν, normal to the outer boundary of the body; ν_P, Poisson coefficient; ρ, partial density of the component; σ, surface tension and stress; φ, degree of saturation of the vapor–gas mixture; Ψ, volume fraction of the porous system occupied by the phase component; ω, specific mass content; Ω, weight parameter of the differential equation; ∇, Hamilton operator. Indices: a, air; g, gas; liq, liquid; v, vapor; ψ = liq, v, a; b, boundary surface of the material; c, parameters inside the capillary; sat, saturated state; eq, equilibrium state; m, outer medium; sol, solid porous body; el, elasticity; 0, initial value; d, diffusion; f, filtration; mean, mean value of the parameter; ef, effective parameters; i, j = 1, 2, 3; m and n, ordinary numbers of the steps of the difference grid for the axes x and t; min and max, minimum and maximum values of the parameter; w, wet; t, total; q, heat flux.

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